

DRAG FORCE IN OPEN CHANNEL

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ABSTRACT

Velocity of flow is more for lesser size of roughness material i.e. for 0.75 inch roughness bed and we know that roughness is more effective in high velocity of flow to raise more depth of water hence more wetted frontal cross sectional area and function of effective roughness concentration depends upon wetted frontal cross sectional area hence function of effective roughness concentration is more for 0.75 inch roughness bed as compared to 1.5 inch roughness bed. Also the size of 0.75 inch roughness bed is lesser than mean depth of flow as compared to 1.5 inch roughness bed hence we get more frontal cross sectional area for 0.75 inch roughness bed hence function of effective roughness concentration is more for 0.75 inch roughness bed.

Subject Headings: Boulders, Channels, Drag Flow Resistance, Flumes

KEYWORDS: Friction Factor, Function of Effective Roughness Concentration, Roughness

INTRODUCTION

High velocity of flow is dominant factor for more function of effective roughness concentration whereas roughness i.e. Darcy Weisbach resistance coefficient, Manning's roughness coefficient are dominant to get more depth of water & more discharge of flow. Hence discharge of flow, mean depth of flow are more for 1.5 inch roughness bed. Also reduction in mean velocity of flow takes place for more roughness.

Experimental Set up and Procedures: Data were obtained for 0.75 inch and 1.5 inch roughness bed.

Flume: The flume is open and 1.168 m wide and 9.54 m long. Each roughness bed was constructed by smearing masonite boards with fiberglass resin. The boards were then screwed to the bed of the flume.

Experimental Procedure: For each bed, five to seven flows were measured for three different slopes (2.5 and 8%). At each flow, depth was gaged at a single cross section, so that mean flow and channel properties could be calculated. In flow with large- scale roughness, the cross- sectional area of flow is significantly affected by the projections of the elements into the flow.

Table 1: Flume Data for 0.75 Inch Roughness Bed

Sl. No.	Channel Slope	Discharge in Cubic Meters per Second	Mean Velocity in Meters per Second	Mean Depth d in Meters
1.	0.02	0.00580	0.222	0.0223
2.	0.02	0.01181	0.348	0.0290
3.	0.02	0.02482	0.484	0.0439
4.	0.02	0.04047	0.586	0.0591
5.	0.02	0.05348	0.656	0.0698
6.	0.05	0.00381	0.230	0.0141

Table 1: Contd.,

7.	0.05	0.00843	0.363	0.0199
8.	0.05	0.02037	0.583	0.0299
9.	0.05	0.03333	0.782	0.0365
10.	0.05	0.04586	0.904	0.0434
11.	0.05	0.05460	0.979	0.0477
12.	0.08	0.00207	0.186	0.0095
13.	0.08	0.00631	0.380	0.0142
14.	0.08	0.01007	0.430	0.0200
15.	0.08	0.02825	0.807	0.0299
16.	0.08	0.04518	1.032	0.0375
17.	0.08	0.04879	1.064	0.0392

Table 2: Flume Data for 0.75 Inch Roughness Bed: $D_{50}=0.013\text{m}$, $D_{84}=0.0193\text{m}$

Sl. No.	Manning's Roughness Coefficient n	Darcy Weisbach Resistance Coefficient f	$\frac{d}{D_{50}}$	$\frac{d}{D_{84}}$
1.	0.071	0.708	1.715	1.155
2.	0.055	0.375	2.231	1.503
3.	0.050	0.294	3.377	2.275
4.	0.051	0.270	4.546	3.062
5.	0.050	0.255	5.369	3.617
6.	0.078	1.046	1.085	0.731
7.	0.065	0.591	1.531	1.031
8.	0.053	0.345	2.300	1.549
9.	0.045	0.234	2.808	1.891
10.	0.043	0.209	3.338	2.249
11.	0.042	0.195	3.669	2.472
12.	0.096	1.731	0.731	0.492
13.	0.063	0.617	1.092	0.736
14.	0.069	0.680	1.538	1.036
15.	0.049	0.289	2.300	1.549
16.	0.043	0.221	2.885	1.943
17.	0.043	0.218	3.015	2.031

Roughness depends upon $\frac{d}{D_{50}}$ and $\frac{d}{D_{84}}$. For large scale roughness $\frac{d}{D_{50}} < 2$ and $\frac{d}{D_{84}} < 1.2$ where d is

the mean depth of flow and D_{50} = the size of the median axis which is bigger than or equal to 50% of median axis.

Similarly D_{84} = The size of the median axis which is bigger than or equal to 84% of median axis. Similarly for Intermediate

Scale roughness $2 < \frac{d}{D_{50}} < 7.5$ and $1.2 < \frac{d}{D_{84}} < 4$.

Table 3: Flume Data for 0.75 Inch Roughness Bed

Sl. No.	Depth d' of Bed Datum in Meters	Relative Roughness Area $\frac{Aw}{Wd'}$
1.	0.0282	0.2081
2.	0.0349	0.1696
3.	0.0495	0.1146
4.	0.0642	0.0801
5.	0.0746	0.0641
6.	0.0204	0.3052

Table 3: Contd.,

7.	0.0262	0.2411
8.	0.0360	0.1709
9.	0.0426	0.1433
10.	0.0491	0.1156
11.	0.0536	0.1090
12.	0.0159	0.4031
13.	0.0211	0.3253
14.	0.0258	0.2222
15.	0.0363	0.1742
16.	0.0435	0.1382
17.	0.0450	0.1285

Table 4: Flume Data for 0.75 Inch Roughness Bed. m=1.696

Sl. No.	Function of Effective Roughness Concentration (b)	$\frac{b}{m}$
1.	0.397	0.234
2.	0.480	0.283
3.	0.660	0.389
4.	0.846	0.499
5.	0.975	0.575
6.	0.269	0.159
7.	0.349	0.206
8.	0.482	0.284
9.	0.560	0.330
10.	0.655	0.386
11.	0.693	0.409
12.	0.189	0.111
13.	0.255	0.150
14.	0.370	0.218
15.	0.477	0.281
16.	0.575	0.339
17.	0.605	0.357

Table 5: Flume Data for 0.75 Inch Roughness Bed

Sl. No.	Hydraulic Radius $R = \frac{A}{P} = \frac{Wd}{W + 2d}$ in Meters
1.	0.021
2.	0.028
3.	0.040
4.	0.054
5.	0.063
6.	0.013
7.	0.019
8.	0.029
9.	0.035
10.	0.041
11.	0.044
12.	0.009
13.	0.014
14.	0.019
15.	0.029
16.	0.035
17.	0.037

Table 6: Flume Data for 1.5 Inch Roughness Bed

Sl. No.	Channel Slope	Discharge in Cubic Meters per Second	Mean Velocity in Meters per Second	Mean Depth d in Meters
1.	0.02	0.00250	0.116	0.0184
2.	0.02	0.00868	0.239	0.0311
3.	0.02	0.01893	0.375	0.0432
4.	0.02	0.04352	0.587	0.0634
5.	0.02	0.06763	0.721	0.0803
6.	0.02	0.08020	0.764	0.0899
7.	0.05	0.00181	0.132	0.0117
8.	0.05	0.00636	0.264	0.0206
9.	0.05	0.01456	0.419	0.0298
10.	0.05	0.03073	0.625	0.0420
11.	0.05	0.06061	0.869	0.0597
12.	0.05	0.07421	0.932	0.0681
13.	0.08	0.00389	0.267	0.0124
14.	0.08	0.01092	0.457	0.0204
15.	0.08	0.02100	0.616	0.0292
16.	0.08	0.03126	0.721	0.0371
17.	0.08	0.05498	0.971	0.0484
18.	0.08	0.05574	0.883	0.0540

Table 7: Flume Data for 1.5 Inch Roughness Bed. $D_{50}=0.034\text{m}$, $D_{84}=0.043\text{m}$

Sl. No.	Manning's Roughness Coefficient n	Darcy Weisbach Resistance Coefficient f	$\frac{d}{D_{50}}$	$\frac{d}{D_{84}}$
1.	0.118	2.135	0.541	0.428
2.	0.084	0.858	0.915	0.723
3.	0.065	0.483	1.271	1.005
4.	0.053	0.288	1.865	1.474
5.	0.050	0.242	2.362	1.867
6.	0.050	0.242	2.644	2.091
7.	0.124	2.650	0.344	0.272
8.	0.092	1.164	0.606	0.479
9.	0.073	0.666	0.876	0.693
10.	0.061	0.422	1.235	0.977
11.	0.054	0.310	1.756	1.388
12.	0.055	0.308	2.003	1.584
13.	0.082	1.093	0.365	0.288
14.	0.067	0.614	0.600	0.474
15.	0.062	0.483	0.859	0.679
16.	0.062	0.448	1.091	0.863
17.	0.054	0.322	1.424	1.126
18.	0.063	0.435	1.588	1.256

Table 8: Flume Data for 1.5 Inch Roughness Bed. $m=1.524$

Sl. No.	Depth d' of Bed Datum in Meters	Relative Roughness Area $\frac{A_w}{Wd'}$	Function of Effective Roughness Concentration b	$\frac{b}{m}$
1.	0.0297	0.3803	0.223	0.153
2.	0.0425	0.2677	0.363	0.238
3.	0.0548	0.2115	0.471	0.309
4.	0.0751	0.1559	0.638	0.419
5.	0.0921	0.1285	0.766	0.503
6.	0.1009	0.1090	0.864	0.567
7.	0.0230	0.4909	0.155	0.102
8.	0.0328	0.3696	0.246	0.161
9.	0.0416	0.2839	0.343	0.225
10.	0.0542	0.2237	0.450	0.295
11.	0.0716	0.1663	0.603	0.396
12.	0.0792	0.1400	0.692	0.454
13.	0.0249	0.5002	0.152	0.100
14.	0.0361	0.4340	0.206	0.135
15.	0.0436	0.3307	0.300	0.197
16.	0.0505	0.2661	0.384	0.252
17.	0.0603	0.1964	0.511	0.335
18.	0.0657	0.1779	0.562	0.369

Table 9: Flume Data for 1.5 Inch Roughness Bed

Sl. No.	Hydraulic Radius in Meters $R = \frac{A}{P} = \frac{Wd}{W + 2d}$ in Meters
1.	0.017
2.	0.030
3.	0.040
4.	0.057
5.	0.071
6.	0.078
7.	0.011
8.	0.020
9.	0.028
10.	0.039
11.	0.054
12.	0.061
13.	0.012
14.	0.020
15.	0.028
16.	0.035
17.	0.045
18.	0.049

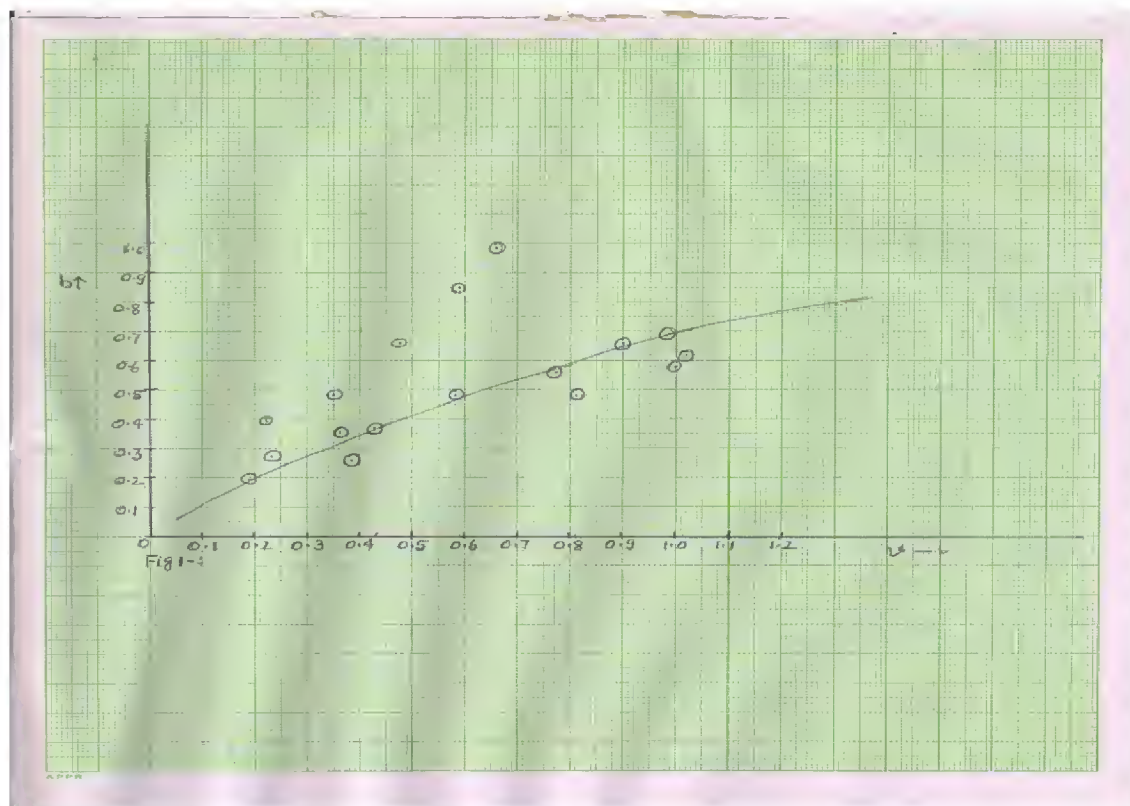


Figure 1: Variation of Parameter Function of Effective Roughness Concentration b with Parameter Mean Velocity of Flow V for 0.75 Inch Roughness Bed

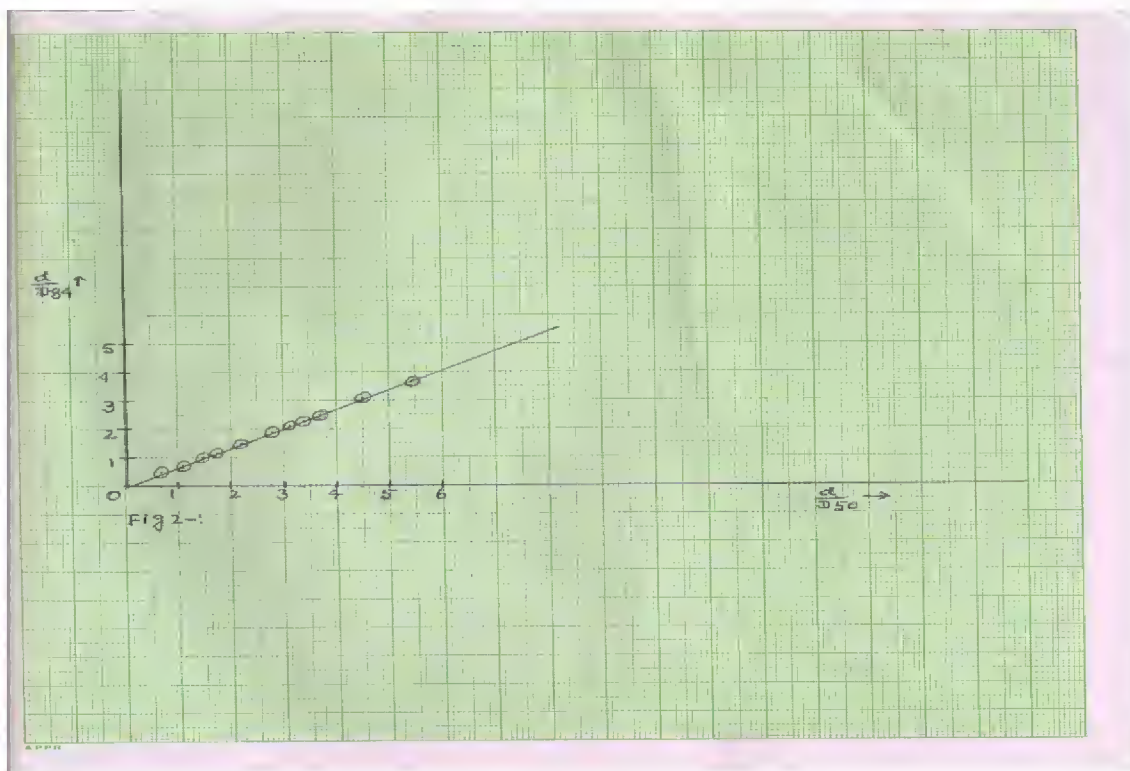


Figure 2: Variation of Parameter $\frac{d}{D_{84}}$ with Parameter $\frac{d}{D_{50}}$ for 0.75 Inch Roughness Bed

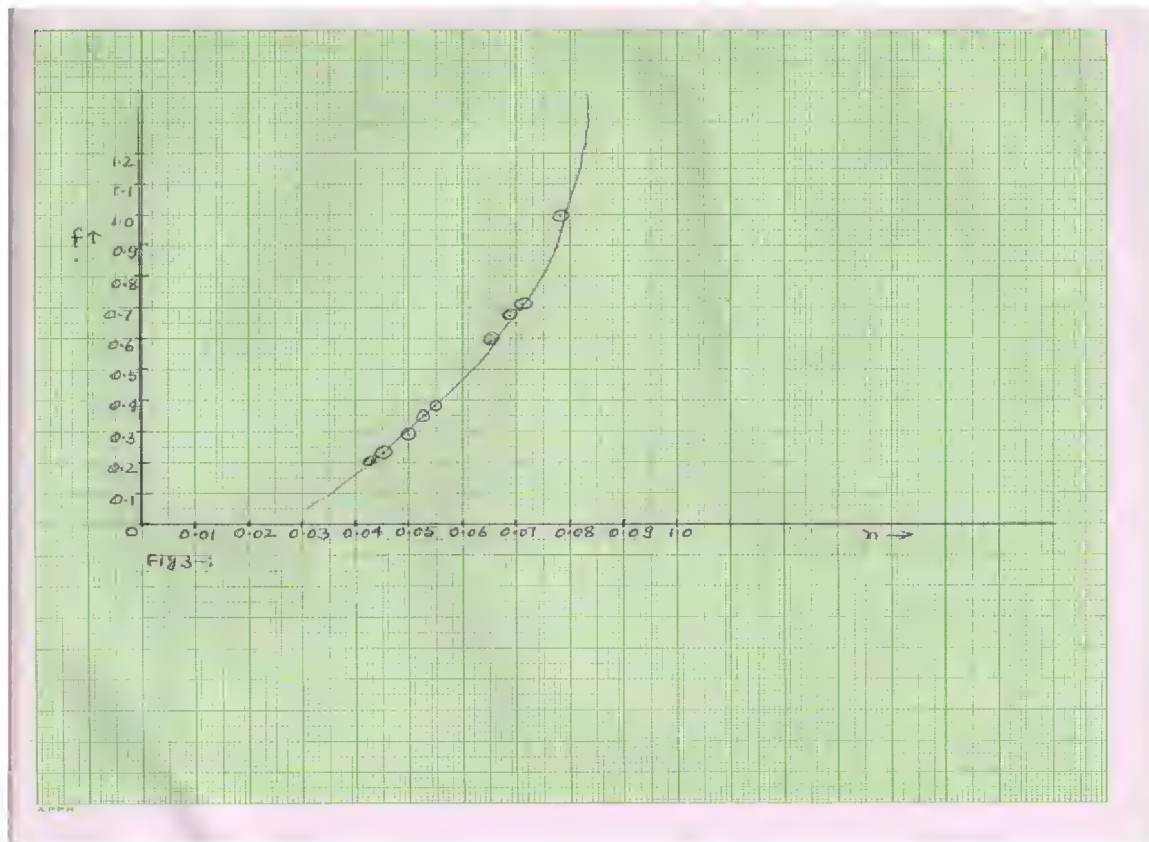


Figure 3: Variation of Parameter Darcy Weisbach Resistance Coefficient f with Parameter Manning's Roughness Coefficient n for 0.75 Inch Roughness Bed

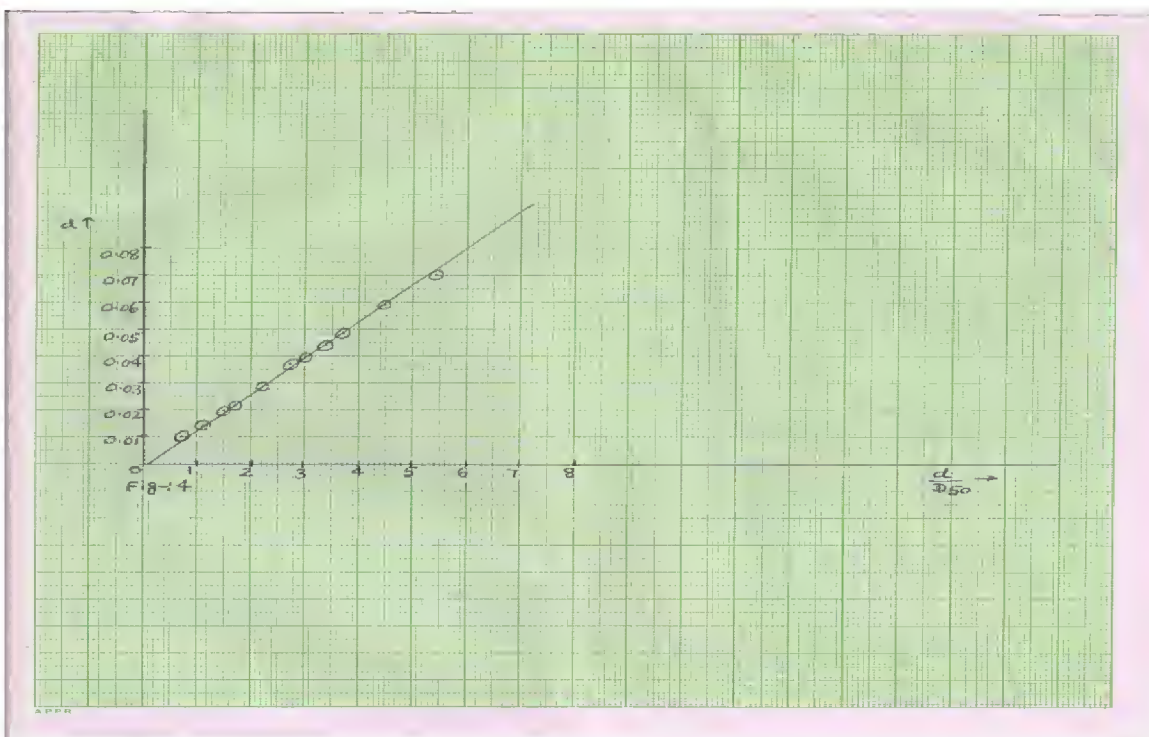


Figure 4: Variation of Parameter Mean Depth of Flow d with Parameter $\frac{d}{D_{50}}$ for 0.75 Inch Roughness Bed

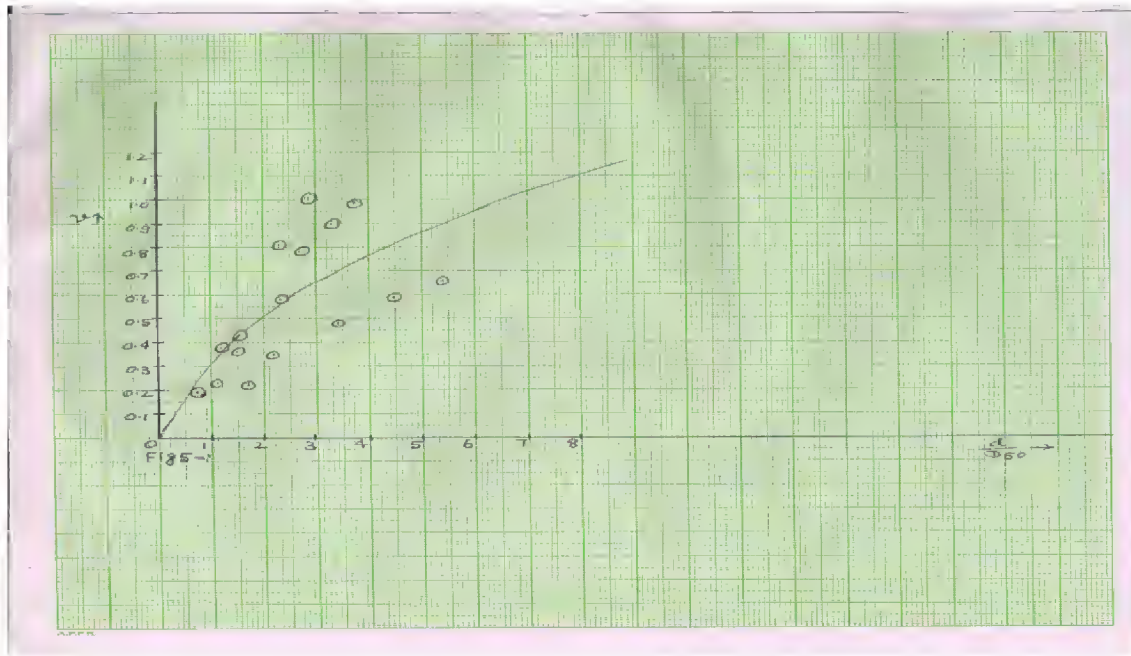


Figure 5: Variation of Parameter Mean Velocity of Flow V with Parameter $\frac{d}{D_{50}}$ for 0.75 Inch Roughness Bed

RESULTS AND ANALYSIS

$$\text{Average } Q = 0.0261 \text{ m}^3/\text{sec}$$

Average value of mean depth of flow $d = 0.0333$ meter.

Average mean velocity of flow $= 0.590$ m/sec.

Average function of effective roughness concentration

$$b = 0.520$$

$$\text{Average } \frac{b}{m} = 0.306$$

$$\text{Average } \frac{d}{D_{50}} = 2.561$$

$$\text{Average } \frac{d}{D_{84}} = 1.725$$

Average value of Manning's roughness coefficient

$$n = 0.057$$

Average value of Darcy Weisbach resistance coefficient $f = 0.487$

1.5 Inch Roughness Bed

Average value of discharge of flow $Q = 0.0326$ m³/sec

Average value of mean velocity of flow $= 0.553$

Average mean depth of flow $d = 0.0422$ meter

Average value of function of effective roughness concentration $b = 0.441$

$$\text{Average value of } \frac{b}{m} = 0.289$$

Average value of $n = 0.071$

Average value of $f = 0.731$

$$\text{Average value of } \frac{d}{D_{50}} = 1.241$$

$$\text{Average value of } \frac{d}{D_{84}} = 0.982$$

Since roughness depends upon size of the roughness material i.e. D_{50} and D_{84} are more for 1.5 inch roughness bed hence lesser value of $\frac{d}{D_{50}}$ and $\frac{d}{D_{84}}$ for 1.5 inch roughness bed which indicates more roughness. Also Manning's roughness coefficient and Darcy Weisbach resistance coefficient are more for 1.5 inch roughness bed which indicates more roughness. Since velocity of flow is more for lesser size of roughness material i.e. for 0.75 inch roughness bed and we know that roughness is more effective in high velocity of flow to raise more depth of water i.e. more wetted frontal cross sectional area and function of effective roughness concentration depends upon wetted frontal cross sectional area hence function of effective roughness concentration b and ratio $\frac{b}{m}$ are more for 0.75 inch roughness bed where $m = \text{constant}$ which depends upon the roughness material hence $\frac{b}{m}$ depends upon b . Hence high velocity is dominant for more function of effective roughness concentration whereas roughness i.e. $(\frac{d}{D_{50}}, \frac{d}{D_{84}}, f, n)$ are dominant to get more depth of water & more discharge of flow hence Q and d are more for 1.5 inch roughness bed. Also reduction in mean velocity of flow takes place for more roughness.

Since larger size of roughness material provides more roughness i.e. D_{84} hence $\frac{d}{D_{84}}$ roughness is more effective i.e. difference in the value of $\frac{d}{D_{50}}$ for two roughness bed is more as compared to hence $\frac{d}{D_{84}}$, hence $\frac{d}{D_{84}}$ is more effective roughness i.e. for lesser value of $\frac{d}{D_{84}}$ we get more d & Q and lesser mean velocity of flow. Similarly difference for n is lesser between 0.75 inch roughness bed and 1.5 inch roughness bed as compared to Darcy Weisbach resistance coefficient hence Manning's roughness coefficient is more effective to get more capacity of the channel as

compared to Darcy Weisbach resistance coefficient i.e. for lesser value of n we get more mean depth of flow and more discharge of flow i.e. more capacity of the channel and also reduction in mean velocity of flow.

0.75 Inch Roughness Bed

Relationship for $Q_{\max}$ with b , $\frac{b}{m}$, $\frac{d}{D_{50}}$, $\frac{d}{D_{84}}$, n & f :-

$$\begin{aligned} Q_{\max} = & 1.839 \left(\frac{d}{D_{50}} \right)^{0.698} - 1.333(b)^{0.750} - 1.337 \left(\frac{b}{m} \right)^{0.748} \\ & - 1.433 \left(\frac{d}{D_{84}} \right)^{0.698} - 0.737(n)^{1.357} \\ & - 0.400(f)^{2.497} + 2.092Q \end{aligned} \quad (1)$$

Relationship for Q with b , $\frac{b}{m}$, $\frac{d}{D_{50}}$, $\frac{d}{D_{84}}$, n & f :-

$$\begin{aligned} Q = & 0.898 \left(\frac{d}{D_{50}} \right)^{1.113} - 0.917(b)^{1.090} - 0.918 \left(\frac{b}{m} \right)^{1.089} \\ & - 0.898 \left(\frac{d}{D_{84}} \right)^{1.114} - 0.860(n)^{1.163} \\ & - 0.593(f)^{1.685} + \frac{Q_{\max}}{2.092} \end{aligned} \quad (2)$$

Relationship for $d_{\max}$ with $\frac{d}{D_{50}}$, b , $\frac{b}{m}$, $\frac{d}{D_{84}}$, n & f :-

$$\begin{aligned} d_{\max} = & 3.326 \left(\frac{d}{D_{50}} \right)^{0.477} - 1.875(b)^{0.533} \\ & - 1.879 \left(\frac{b}{m} \right)^{0.532} - 2.097 \left(\frac{d}{D_{84}} \right)^{0.477} \\ & - 0.877(n)^{1.140} - 0.524(f)^{1.190} + 2.096d \end{aligned} \quad (3)$$

Relationship for d with $\frac{d}{D_{50}}$, b , $\frac{b}{m}$, $\frac{d}{D_{84}}$, n & f :-

$$d = 0.948 \left(\frac{d}{D_{50}} \right)^{1.113} - 0.927(b)^{1.079} - 0.928 \left(\frac{b}{m} \right)^{1.077} - 0.898 \left(\frac{d}{D_{84}} \right)^{1.114} - 0.930(n)^{1.075} - 0.708(f)^{1.412} + \frac{d_{\max}}{2.096} \quad (4)$$

Relationship of V with $\frac{d}{D_{50}}$, b , $\frac{b}{m}$, $\frac{d}{D_{84}}$, n & f :-

$$V = 2.632 \left(\frac{d}{D_{50}} \right)^{0.563} - 1.627(b)^{0.615} - 1.631 \left(\frac{b}{m} \right)^{0.613} - 1.775 \left(\frac{d}{D_{84}} \right)^{0.563} - 0.895(n)^{1.118} - 0.554(f)^{1.804} + \frac{V_{\max}}{1.803} \quad (5)$$

Relationship of $V_{\min}$ with $\frac{d}{D_{50}}$, b , $\frac{b}{m}$, $\frac{d}{D_{84}}$, n & f :-

$$V_{\min} = 0.740 \left(\frac{d}{D_{50}} \right)^{1.493} - 0.763(b)^{1.310} - 0.765 \left(\frac{b}{m} \right)^{1.308} - 0.670 \left(\frac{d}{D_{84}} \right)^{1.494} - 1.246(n)^{0.803} - 1.454(f)^{0.688} + \frac{V_{\max}}{4.793} \quad (6)$$

Relationship of $\frac{d}{D_{50}}$ with b , $\frac{b}{m}$, $\frac{d}{D_{84}}$, n & f :-

$$\frac{d}{D_{50}} = 2.221 \left(\frac{d}{D_{84}} \right)^{0.912} - 1.077(b)^{0.930} - 1.078 \left(\frac{b}{m} \right)^{0.927} - 0.789(n)^{1.267} - 0.480(f)^{2.081} \quad (7)$$

Relationship of $\frac{d}{D_{50}}$ with b , $\frac{b}{m}$:-

$$\frac{d}{D_{50}} = 5.369(b)^{0.930} - 1.078 \left(\frac{b}{m} \right)^{0.927} \quad (8)$$

Relationship of $\frac{d}{D_{50}}$ with n & f :-

$$\frac{d}{D_{50}} = 11.583 (f)^{2.081} - 0.789 (n)^{1.267} \quad (9)$$

Relationship of $\frac{d}{D_{50}}$ with $\frac{d}{D_{84}}$:-

$$\frac{d}{D_{50}} = 1.557 \left(\frac{d}{D_{84}} \right)^{0.912} \quad (10)$$

Mathematical Formulation for $Q_{\max}$

$$\begin{aligned} Q_{\max} &= 1.839 \left(\frac{d}{D_{50}} \right)^{0.698} - 1.333 (b)^{0.750} \\ &- 1.337 \left(\frac{b}{m} \right)^{0.748} - 1.433 \left(\frac{d}{D_{84}} \right)^{0.698} \\ &- 0.737 (n)^{1.357} - 0.400 (f)^{2.497} + 2.092 Q \end{aligned} \quad (1)$$

$$\begin{aligned} Q &= 0.898 \left(\frac{d}{D_{50}} \right)^{1.113} - 0.917 (b)^{1.090} \\ &- 0.918 \left(\frac{b}{m} \right)^{1.089} - 0.898 \left(\frac{d}{D_{84}} \right)^{1.114} \\ &- 0.860 (n)^{1.163} - 0.593 (f)^{1.685} + \frac{Q_{\max}}{2.092} \end{aligned} \quad (2)$$

Substituting Q from in (2) in (1) we get

$$\begin{aligned} Q_{\max} &= \text{Zero} + 2.092 \left[0.898 \left(\frac{d}{D_{50}} \right)^{1.113} - 0.917 (b)^{1.090} - 0.918 \left(\frac{b}{m} \right)^{1.089} \right. \\ &- 0.898 \left(\frac{d}{D_{84}} \right)^{1.114} - 0.860 (n)^{1.163} - 0.593 (f)^{1.685} + \frac{0.05460}{2.092} \left. \right] \\ &= 2.092 [2.558 - 0.450 - 0.253 - 1.648 - 0.031 - 0.176 + 0.0261] \\ &= 2.092 [2.584 - 2.558] = 0.0544 \approx 0.05460 \text{ m}^3 / \text{sec} \end{aligned}$$

Hence equation is satisfied.

Mathematical formulation for Q

Substituting $Q^{\max}$ from (1) in equation (2) we get,

$$\begin{aligned}
 Q &= \text{Zero} + \frac{1}{2.092} \left[1.839 \left(\frac{d}{D_{50}} \right)^{0.698} - 1.333(b)^{0.750} - 1.337 \left(\frac{b}{m} \right)^{0.748} \right. \\
 &\quad \left. - 1.433 \left(\frac{d}{D_{84}} \right)^{0.698} - 0.737(n)^{1.357} - 0.400(f)^{2.497} + 2.092Q \right] \\
 \text{Hence } Q &= \frac{1}{2.092} [3.545 - 0.816 - 0.551 - 2.097 - 0.015 - 0.066 + 0.055] \\
 &= \frac{1}{2.092} [3.600 - 3.545] \\
 &= 0.263 \text{ m}^3/\text{sec} \approx 0.0261 \text{ m}^3/\text{sec}
 \end{aligned}$$

Hence equation is satisfied.

Mathematical formulation for $d_{\max}$:-

$$\begin{aligned}
 d_{\max} &= 3.326 \left(\frac{d}{D_{50}} \right)^{0.447} - 1.875(b)^{0.533} - 1.879 \left(\frac{b}{m} \right)^{0.532} - 2.097 \left(\frac{d}{D_{84}} \right)^{0.477} \\
 &\quad - 0.877 (n)^{1.140} - 0.524 (f)^{1.910} + 2.096 d
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 d &= 0.948 \left(\frac{d}{D_{50}} \right)^{1.113} - 0.927(b)^{1.079} - 0.928 \left(\frac{b}{m} \right)^{1.077} - 0.898 \left(\frac{d}{D_{84}} \right)^{1.114} \\
 &\quad - 0.930 (n)^{1.075} - 0.708(f)^{1.412} + \frac{d_{\max}}{2.096}
 \end{aligned} \tag{2}$$

Substituting d from equation (2) in equation (1)

$$\begin{aligned}
 d_{\max} &= 3.326 \left(\frac{d}{D_{50}} \right)^{0.447} - 1.875(b)^{0.533} - 1.879 \left(\frac{b}{m} \right)^{0.532} - 2.097 \left(\frac{d}{D_{84}} \right)^{0.477} \\
 &\quad - 0.877 (n)^{1.140} - 0.524 (f)^{1.910} + 2.096 \left[0.948 \left(\frac{d}{D_{50}} \right)^{1.113} - 0.927 (b)^{1.079} - 0.928 \left(\frac{b}{m} \right)^{1.077} \right. \\
 &\quad \left. - 0.898 \left(\frac{d}{D_{84}} \right)^{1.114} - 0.930 (n)^{1.075} - 0.708 (f)^{1.412} + \frac{d_{\max}}{2.096} \right]
 \end{aligned}$$

$$\begin{aligned}
&= 5.209 - 1.323 - 1.001 - 2.720 - 0.033 - 0.133 + 2.096 [2.700 - 0.458 - 0.259 - 1.648 - 0.043 - \\
&\quad - 0.256 + 0.033] \\
&= 5.209 - 5.210 + 2.06 [2.733 - 2.664] \\
&= 5.209 - 5.210 + 0.145
\end{aligned}$$

Hence $d_{\max} = 0.144 \text{ meter} \approx 0.0698 \text{ meter}$

Hence equation is satisfied .

Mathematical formulation for mean depth of flow (d):

Substituting $d_{\max}$ from equation (1) in equation (2) we get:

$$\begin{aligned}
d &= 0.948 \left(\frac{d}{D_{50}} \right)^{1.113} - 0.927(b)^{1.079} - 0.928 \left(\frac{b}{m} \right)^{1.077} - 0.898 \left(\frac{d}{D_{84}} \right)^{1.114} \\
&\quad - 0.930(n)^{1.075} - 0.708(f)^{1.412} + \frac{1}{2.096} \left[\left(\frac{d}{D_{50}} \right)^{0.477} - 1.875(b)^{0.533} - 1.879 \left(\frac{b}{m} \right)^{0.532} \right. \\
&\quad \left. - 2.097 \left(\frac{d}{D_{84}} \right)^{0.477} - 0.877(n)^{1.140} - 0.524(f)^{1.910} + 2.096d \right]
\end{aligned} \tag{3}$$

We know from equation (7)

$$\begin{aligned}
\frac{d}{D_{50}} &= 2.211 \left(\frac{d}{D_{84}} \right)^{0.912} - 1.077(b)^{0.930} - 1.078 \left(\frac{b}{m} \right)^{0.927} - 0.789(n)^{1.267} - 0.480(f)^{2.081} \\
&= 3.635 - 0.586 - 0.360 - 0.021 - 0.107 \\
&= 3.635 - 1.074 = 2.261
\end{aligned}$$

From equation (8) we know:

$$\begin{aligned}
\frac{d}{D_{50}} &= 5.369(b)^{0.930} - 1.078 \left(\frac{b}{m} \right)^{0.927} \\
&= 2.923 - 0.360 \\
&= 2.563
\end{aligned}$$

From equation (9) we know:

$$\begin{aligned}
\frac{d}{D_{50}} &= 11.583(f)^{2.081} - 0.789(n)^{1.267} \\
&= 2.592 - 0.021
\end{aligned}$$

$$= 2.571$$

From equation (10) we know

$$\frac{d}{D_{50}} = 1.557 \left(\frac{d}{D_{84}} \right)^{0.912}$$

$$= 2.560$$

Hence comparing equation (7), (8), (9) and (10) the most appropriate value of $\frac{d}{D_{50}} = 2.561$ which is obtained

from equation (7) which is equal to average value of $\frac{d}{D_{50}}$ equation (7) is appropriate for $\frac{d}{D_{50}} = 2.561$. Hence we take

$\frac{d}{D_{50}}$ from equation (7) and substitute in equation (3).

Hence from equation (3) we get

$$d = 2.700 - 0.458 - 0.259 - 1.648 - 0.043 - 0.256 + \frac{1}{2.096} [5.209 - 1.323 - 1.001$$

$$- 2.720 - 0.033 - 0.133 + 0.055]$$

$$d = 2.700 - 0.664 + \frac{1}{2.096} [5.264 - 5.210]$$

Hence $d = 0.036 + 0.026$

$$= 0.062 \text{ meter}$$

$$\approx 0.0333 \text{ meter}$$

Hence equation is satisfied.

Mathematical formulation for V:-

$$V = 2.632 \left(\frac{d}{D_{50}} \right)^{0.563} - 1.627(b)^{0.615} - 1.631 \left(\frac{b}{m} \right)^{0.613} - 1.775 \left(\frac{d}{D_{84}} \right)^{0.563} \\ - 0.895(n)^{1.118} - 0.554(f)^{1.804} + \frac{V_{\max}}{1.803} \quad (1)$$

$$V_{\min} = 0.222 \text{ m/sec.}$$

$$V_{\max} = 1.064 \text{ m/sec.}$$

$$\text{Or } 0.222x = 1.064$$

$$\therefore x = 4.793$$

Hence $V_{\max} = 4.793 V_{\min}$

$$V_{\min} = 0.740 \left(\frac{d}{D_{50}} \right)^{1.493} - 0.763(b)^{1.310} - 0.765 \left(\frac{b}{m} \right)^{1.308} - 0.670 \left(\frac{d}{D_{84}} \right)^{1.494} - 1.246(n)^{0.803} - 1.454(f)^{0.688} + \frac{V_{\max}}{4.793} \quad (2)$$

Substituting $V_{\min}$ from (2) in equation (1) we get:-

$$V = 2.632 \left(\frac{d}{D_{50}} \right)^{0.563} - 1.627(b)^{0.615} - 1.631 \left(\frac{b}{m} \right)^{0.613} - 1.775 \left(\frac{d}{D_{84}} \right)^{0.563} - 0.895(n)^{1.118} - 0.554(f)^{1.804} + \frac{4.793}{1.803} \left[0.740 \left(\frac{d}{D_{50}} \right)^{1.493} - 0.763(b)^{1.310} - 0.765 \left(\frac{b}{m} \right)^{1.308} - 0.670 \left(\frac{d}{D_{84}} \right)^{1.494} - 1.246(n)^{0.803} - 1.454(f)^{0.688} + \frac{V_{\max}}{4.793} \right]$$

$$= 4.469 - 1.088 - 0.789 - 2.413 - 0.036 - 0.151 + 2.658 [3.013 - 0.324 - 0.163 - 1.513 - 0.125 -$$

$$0.886 + 0.222]$$

$$= 4.469 - 4.447 + 2.658 [3.235 - 3.011]$$

$$= 0.008 + 0.595$$

$$= 0.587 \text{ m/sec}$$

Hence equation is satisfied ≈ 0.590 m/sec.

Mathematical formulation for $V_{\min}$:-

$$V_{\min} = 0.740 \left(\frac{d}{D_{50}} \right)^{1.493} - 0.763(b)^{1.310} - 0.765 \left(\frac{b}{m} \right)^{1.308} - 0.670 \left(\frac{d}{D_{84}} \right)^{1.494} - 1.246(n)^{0.803} - 1.454(f)^{0.688} + \frac{1.803}{4.793} \left[2.632 \left(\frac{d}{D_{50}} \right)^{0.563} - 1.627(b)^{0.615} - 1.631 \left(\frac{b}{m} \right)^{0.613} \right]$$

$$\left[-1.775 \left(\frac{d}{D_{84}} \right)^{0.563} - 0.895(n)^{1.118} - 0.554(f)^{1.804} + \frac{V_{\max}}{1.803} \right]$$

Substituting V in $V_{\min}$ expression taking

$$V_{\max} = 1.803V$$

$$\text{Now } V_{\min} = 3.013 - 0.324 - 0.163 - 1.513 - 0.125 - 0.886 + 0.376 [4.469 - 1.088 - 0.789 - 2.413 - 0.036 - 0.151 + 0.590]$$

$$= 3.013 - 3.011 + 0.376[5.059 - 4.477]$$

$$= 3.013 - 3.011 + 0.219$$

$$= 0.221 \text{ m/sec.}$$

$$\approx 0.222 \text{ m/sec.}$$

Hence equation is satisfied.

CONCLUSIONS

Since larger size of roughness material provides more roughness i.e. D_{84} hence $\frac{d}{D_{84}}$ roughness is more effective i.e. difference in the value of $\frac{d}{D_{50}}$ for two roughness bed is more as compared to $\frac{d}{D_{84}}$ hence $\frac{d}{D_{84}}$ is more effective roughness i.e. for lesser value of $\frac{d}{D_{84}}$ we get more d, Q and lesser mean velocity of flow similarly Manning's roughness co-efficient is more effective parameter as compared to Darcy Weisbach resistance coefficient i.e. for lesser value of n we get more d, Q and also more reduction in mean velocity of flow.

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APPENDICES

The following symbols are used in this paper:

A = Flow cross sectional area

$\frac{A_w}{Wd'}$ = Relative roughness area

A_w = Wetted cross section area

b = Function of effective roughness concentration

d = Mean depth of flow in meters

d' = Depth of bed datum in meters.

D_{50} = The size of median axis which is bigger than or equal to 50% of median axis.

D_{84} = The size of median axis which is bigger than or equal to 84% of median axis.

f = Darcy Weisbach resistance coefficient

g = Acceleration due to gravity

m = Constant varying with bed material properties.

n = Manning's roughness coefficient

P = Wetted parameter

Q = Discharge in cubic meters per second

R = Hydraulic radius = $\frac{A}{p}$

R = $\frac{Wd}{W + 2d}$

S	=	Channel slope
V	=	Mean velocity of flow in meters per second.
W	=	Width of the channel = 1.168m

Formula Used

$$\frac{A_w}{Wd'} = \left(\frac{w}{d}\right)^{-b}$$

$$W = \text{Width of the channel} = 1.168\text{m}$$

$$\left(\frac{8}{f}\right)^{\frac{1}{2}} = \frac{V}{(gRS)^{\frac{1}{2}}}$$

$$R = \text{Hydraulic radius} = \frac{Wd}{W + 2d}$$

$$V = \frac{1.49}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$m = 1.025 \left(\frac{W}{Y_{50}}\right)^{0.118}$$

$$W = \text{Width of the channel} = 1.168 \text{ m}$$

$$Y_{50} = \text{The cross stream axis is assumed to have been the average of the long } L_{50} \text{ and median axis } D_{50} \text{ for 0.75 inch roughness bed}$$

$$Y_{50} = 0.0164\text{m}$$

$$W = 1.168\text{m}$$

$$\text{Hence } m = 1.696$$

$$A + A_w = Wd'$$

$$A = \text{Flow cross sectional area}$$

$$A_w = \text{Wetted cross sectional area}$$

For 1.5 inch roughness bed:

$$Y_{50} = 0.0405\text{m}$$

$$m = 1.025 \left(\frac{W}{Y_{50}}\right)^{0.118} \quad \text{Hence } m = 1.524$$

